

## The Proof for Multiplying on your Fingers

*(This trick is often called "Sluggard's Rule" and has been around since the late 1500s)*

This is a pretty great trick! Let's see if we can figure out why it works. The first step is to try to describe exactly what we are doing. When we want to multiply two numbers, how many fingers will we have up and down on each hand? We could quickly figure this out for any particular number, but in order to show that this technique will work for every problem we throw at it, we are going to want something a bit more general. What we want is a formula for how many fingers will be up, and how many will be down, for *any* number from 6 to 10 that we choose.

I think the easiest way to figure this formula out is to hold up both hands, with palms down and fingers curled down. Now start uncurling your fingers one by one, starting with your left pinky and moving from left to right.



*With six fingers raised, notice that the right hand is in position for the finger trick: everything from the thumb up to the six finger (the thumb) is raised, while everything after that is curled down.*



*With eight fingers raised, notice that the right hand is in position for the finger trick: everything from the thumb up to the eight finger (the middle finger) is raised, while everything after that is curled down.*

Now, we are only looking at the numbers 6 through 10, so let's pick some number  $n$  in this range and see what happens when we have lifted this many fingers. Notice that our left hand is completely up, and our right hand looks exactly like it should, with fingers up to and including the  $n$  finger raised, and with everything after it, out to the pinky, curled! So how many fingers are raised on the right hand? There are  $n$  total fingers raised, but five are on the left hand, so

$$\text{number up} = n - 5.$$



And how many will be curled? Well, of our ten fingers, all but  $n$  of them are curled down, and they are all on the right hand. Thus,

$$\text{number curled} = 10 - n.$$

Awesome! So now when we multiply two numbers,  $n * m$ , we touch the  $n$  finger on the left with the  $m$  finger on the right and curl everything towards the pinky. Now, when we count the raised fingers by ten, we get

$$\begin{aligned} 10 * \text{total number up} &= 10 * (\text{number up on left hand} + \text{number up on right hand}) \\ &= 10(n - 5 + m - 5) \\ &= 10n + 10m - 100 \end{aligned}$$

and when we multiply the number of fingers curled, we get

$$\begin{aligned} \text{number curled on left hand} * \text{number curled on right hand} &= (10 - n)(10 - m) \\ &= 100 - 10n - 10m + nm \end{aligned}$$

I'll bet you already see where this is going.

When we add these two parts together, and wait for all the dust to settle...

$$\begin{aligned} &= (10n + 10m - 100) + (100 - 10n - 10m + nm) \\ &= nm \end{aligned}$$

So we see that for any  $n$  and  $m$ , this finger trick will give us exactly the product we want.

Now think about this:

- What if we had 14 fingers, 7 on each hand? Explain whether or not the trick would still work, and if not, what you would have to do to fix it. What range of multiplication problems would it work for?
- What if we had 4 fingers on one hand, and 7 on the other? What happens then?

- Zach Gaslowitz

