Divisibility Rules

When we say a number is divisible by another number, we mean that if we divide a whole number by another whole number that the result will be a whole number.
For example, 267 is divisible by 9 since $267 \div 9 = 29$.

For many numbers it is possible to test for divisibility without actually dividing them.

Some divisibility tests can be done by looking at the ones digit.
A number is:

- divisible by 2 → if the ones digit is even (2,4,6,8,0)
- divisible by 5 → if the ones digit is 5 or 0
- divisible by 10 → if the ones digit is 0

Some divisibility tests can be done by adding all the digits together.
A number is:

- divisible by 3 → if the sum of the digits is divisible by 3
- divisible by 9 → if the sum of the digits is divisible by 9

ex. 87 is divisible by 3 because 8 + 7 = 15 and 1 + 5 = 6 and 6 is divisible by 3
ex. 261 is divisible by 9 because 2 + 6 + 1 = 9 and 9 is divisible by 9

Some less common divisibility tests.
A number is:

- divisible by 4 → if the last two digits are divisible by 4
- divisible by 8 → if the last three digits are divisible by 8

- divisible by 6 → if it follows the rules for 2 and 3
- divisible by 12 → if it follows the rules for 3 and 4

ex. 432 is divisible by 3 because 4 + 3 + 2 = 9 and 9 is divisible by 3
432 is divisible by 4 because 32 is divisible by 4
therefore 432 is divisible by 12

- divisible by 11 → if when you add and subtract digits in an alternating pattern, the answer is divisible by 11.

ex. 1,012 is divisible by 11 because +1 – 0 + 1 – 2 = 0 and 0 is divisible by 11
ex. 616 is divisible by 11 because +6 – 1 + 6 = 11 and 11 is divisible by 11

Divisibility Rules

Some even rarer divisibility tests.

A number is:

- **divisible by 7** → First double the last digit and subtract it from the truncated number made by the remaining digits.
  If that answer is divisible by 7 then the original number was divisible by 7.
  (This process can be repeated.)

ex. 987 → the last digit is 7 doubled is 14 → 98 – 14 = 84 → 84 is divisible by 7, so 987 is divisible by 7

ex. 62,216 →

a. 62,216	*b. 6209*	*c. 602*
the last digit is 6 doubled is 12	*the last digit is 9 doubled is 18*	*the last digit is doubled is 4*
6221 – 12 = 6209	*620 – 18 = 602*	*60 – 4 =56*

e. 56 is divisible by 7, so 62,216 is divisible by 7

- **divisible by 13** → First multiply the last digit by 4 and add it to the truncated number made by the remaining digits.
  If that answer is divisible by 13 then the original number was divisible by 13.

ex. 182 → the last digit is 2, 4 time 2 is 8 → 18 + 8 = 26 → 26 is divisible by 13, so 182 is divisible by 13

ex. 8,450 →

a. 8450	*b. 845*	*c. 104*
the last digit is 0 times 4 = 0	*the last digit is 5 times 4 = 20*	*the last digit is 4 times 4 = 16*
845 + 0 = 845	*84 + 20 = 104*	*10 + 16 = 26*

e. 26 is divisible by 13, so 8,450 is divisible by 13.

- **divisible by 17** → First multiply the last digit by 5 and subtract it from the truncated number made by the remaining digits.
  If that answer is divisible by 17 then the original number was divisible by 17.

ex. 867 → the last digit is 7, 5 time 7 is 35 → 86 - 35 = 51 → 51 is divisible by 17, so 867 is divisible by 17

Sources:
Divisibility by prime numbers under 50.© Stu Savory, 2003 & 2004. <http://www.savory.de/maths1.htm>
Su, Francis E., et al. "Divisibility by Eleven." *Math Fun Facts*. <http://www.math.hmc.edu/funfacts>.