The Magic of Six

The Math:

Pick a number between 1 and 52. ____________

Multiply that number by 3. This will give you the value of A.

A = _________________

Subtract one from A to get B. B = _________________

Subtract one from B to get C. C = _________________

Add A, B and C. A + B + C = _________________

Add together each digit of your new answer until you have only one digit.

Set-up:

- Take a deck of cards, and put a 6 on the top of the deck.
- Have pencil, or a calculator, and paper for the person to solve the problem on.

As a magic trick:

Take the deck of cards and shuffle or mix them throughout the trick (always leaving the 6 on the top of the deck).

Have the person do the math.

When they get their final 1 digit number, tell them:

> "I think your number has mysteriously risen to the top of the deck."

Flip over the top card on the deck (the 6)

Why it works:

So once you do the first few steps you get $(3n) + (3n - 1) + (3n - 2) = 9n - 3$.

Well, 9n must be divisible by 9, and the digits of a number that is divisible by 9 add up to 9. (Remember this from your divisibly rules. (ex. 81 → 8 + 1 = 9, 45 → 4 +5 = 9))

So the digits of 9n add up to 9, and 9 – 3 is 6.

The real proof requires a basic understanding of modular arithmetic:

Modular arithmetic is like counting on a clock (mod 12), every time you reach the mod number you return to 0.

The mod of a number is the remainder if you were to divide that number by the mod (ex. 19 mod 9 is 1 since 19 divided by 9 is 2 with a remainder of 1).

Zach's Real Proof:

To understand why this trick works you first have the amazing insight that a number and the sum of its digits are always congruent mod 9. (Abstract Algebra Theorem)

The best way to see that is to take a number - 256, say. Write it as $2 * 10^2 + 5 * 10 + 6$, and look at it mod 9, and it will still be congruent to 256, but every power of 10 is congruent to 1 mod 9, so this just adding up the digits. So 256 is congruent mod 9 to 2 + 5 + 6 = 13, which 1 + 3 = 4.

256 mod 9 = 4 (since $256 \div 9 = 28R4$)

9n – 3 is congruent to mod 9 – 3, and thus 6. But on the other hand, it is also congruent to the sum of the digits, or the sum of those digits until you get down to a single digit, and since it has to be congruent to 6, the number will be exactly 6.

(see "Why does the 'dividing by 9 rule work?")

Why does the 'divisibility by 9' rule work?

The only way that I can think of to simply explain this would be as follows:

Look at a 2 digit number: $10a + b = 9a + (a + b)$.

We know that $9a$ is divisible by 9, so $10a + b$ will be divisible by 9
if and only if $a + b$ is.

Similarly, $100a + 10b + c = 99a + 9b + (a + b + c)$, and $99a + 9b$ is divisible by 9, so the total will be if $a + b + c$ is.

This explanation also works to prove the divisibility by 3 test.

It clearly originates from modular arithmetic ideas, and I'm not sure if it's simple enough, but it's the only explanation I can think of.

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