

Mathematicians try to understand the patterns in the world. When they look at the natural numbers (1, 2, 3 . . .) they notice that one way the numbers can be organized was into two types of numbers: prime numbers and composite numbers. A prime number has exactly 2 factors 1 and itself, for example 11, is a prime number because the only numbers that divide equally into it are 1 and 11. Composite numbers have more than 2 factors, for example 4, is a composite number because 1, 2, and 4 can divide into it without having a remainder. One is not considered to be prime or composite since it has exactly 1 factor.

Different types of primes and some of what has been learned about prime numbers:

1. Around 300 BCE, Euclid proved that there are infinitely many prime numbers and that every natural number greater than 1 is either prime or can be written as a product of primes.

Q. Write 80 as a product of primes.

Non-prime Number	Written as a product of primes.
4	$2 * 2$
18	$2 * 3 * 3$
80	

2. Paul Stäckel (1862-1919) called prime numbers that are separated by just one composite number, twin primes. They are of the form $(p, p+2)$, 3 and 5 are an example of twin primes.

Q. Find at least three other examples of twin primes.

3. Albert Girard (1595 to 1632 - France) speculated that every prime number of the form $4n + 1$ can be written as the sum of two squares. (Fermat later proved this.)

Q. Complete the table below:

n =	$4n + 1$	Is it prime?	Written as a sum of two squares
1	$4 * 1 + 1 = 5$	yes	$1^2 + 2^2 = 1 + 4 = 5$
2	$4 * 2 + 1 = 9$	no	
3			
4			

4. Christian Goldbach(1690-1764 Russia) conjectures:

a. That every even number greater than 2 can be written as the sum of two prime numbers. While this has not been proven, it has been checked for every number up to $4 * 10^{18}$.

b. That every odd number greater than 5 can be written as the sum of three prime numbers.

a. Give 2 more examples

Even number greater than 2	Written as a sum of 2 prime numbers
20	13 + 7
30	11 + 19

b. Give 2 more examples

Odd number greater than 5	Written as a sum of 3 prime numbers
7	2 + 2+ 3
9	

5. Sophie Germain (1776 to 1831 France)

Sophie Germain started studying math at the age of 13 by reading books in her father’s library. She loved math so much that she even taught herself Latin and Greek so she could read the works of past and current mathematicians. At first her parents did not approve of her studying math. They believed, like so many at that time, that women could not handle the complexities of advanced mathematics. To prevent her parents from finding out, she studied at night. When they found out anyway, they denied her a fire for her bedroom and warm clothes when she went to bed, but when that did not work they relented and supported her in her studies. Germain corresponded with many of the great mathematicians alive at that time. While Germain was denied access to The Academy of Math and Science, her work helped to lead the way for women to be admitted. Sophie Germain died at the age of 55. Shortly before this Gauss, one of her earliest mentors, had convinced the University of Gottingen to give Germain an honorary degree. She died before she could receive it.

A Sophie Germain Prime number is a prime number that, when you multiply it by 2 and add 1, you get another prime number. (The prime number of the form $2n+1$ are called safe primes.) These primes are often used in cryptography.

Q. Fill out the table below:

Prime number n	$2n + 1$	Is $2n + 1$ a prime number?	If yes, then n is a Sophie Germain prime	If yes, then $2n + 1$ is a Safe Prime
29	$2 * 29 + 1 = 59$	yes	29	59
2	$2 * 2 + 1 = 5$	yes	2	5
3				
11				

6. Mersenne primes: are prime numbers that can be found by the formula $2^p - 1$ (where p is a prime number). This formula does not work for all values of p , but as of January 2016, eleven of the twelve largest known primes were Mersenne primes. The largest known prime is $2^{74207281} - 1$, and has 22,338,618 digits.

Q. Fill out the table below:

$p =$	$2^p - 1$	Is this prime?
2	$2^2 - 1 = 4 - 1 = 3$	yes
3		
5		

Q. If we were to write out the largest known prime, how many pages would it take? (A page holds about 3,000 characters.)

7. Perfect numbers are numbers where all the factors, except itself, add up to that number.
Ex. The factors of 6 are 1, 2, 3, 6 and $1 + 2 + 3 = 6$

Euclid (around 300 BCE) showed that if the number $2^n - 1$ is prime (see Mersenne primes) then the number $2^{n-1}(2^n - 1)$ is a perfect number. Euler (in 1747) proved that all even perfect numbers are of this form. It is not known whether there are any odd perfect numbers.

Q. Find a perfect number other than 6. How did you find that number? How do you know it is a perfect number?

Sources:

"Goldbach's conjecture" and "Sieve of Eratosthenes" <http://primes.utm.edu/glossary/>

"Prime numbers" (May 2009) http://www-history.mcs.st-and.ac.uk/HistTopics/Prime_numbers.html

Weisstein, Eric W. "Twin Primes." From *MathWorld*--A Wolfram Web Resource.
<http://mathworld.wolfram.com/TwinPrimes.html>

"Sophie Germain" (accessed on August 28, 2016) https://en.wikipedia.org/wiki/Sophie_Germain

"Read Euler, read Euler, he is the master of us all." Robin Wilson (March 1, 2007)
<https://plus.maths.org/content/os/issue42/features/wilson/index>

The Primes From 1 to 1,000

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97

How many prime numbers are there between 1 and 100?

101, 103, 107, 109, 113, 127, 131, 137, 139, 149, 151, 157, 163, 167, 173, 179, 181, 191, 193, 197, 199

How many prime numbers are there between 101 and 200?

211, 223, 227, 229, 233, 239, 241, 251, 257, 263, 269, 271, 277, 281, 283, 293

How many prime numbers are there between 201 and 300?

307, 311, 313, 317, 331, 337, 347, 349, 353, 359, 367, 373, 379, 383, 389, 397

How many prime numbers are there between 301 and 400?

401, 409, 419, 421, 431, 433, 439, 443, 449, 457, 461, 463, 467, 479, 487, 491, 499

How many prime numbers are there between 401 and 500?

503, 509, 521, 523, 541, 547, 557, 563, 569, 571, 577, 587, 593, 599

How many prime numbers are there between 501 and 600?

601, 607, 613, 617, 619, 631, 641, 643, 647, 653, 659, 661, 673, 677, 683, 691

How many prime numbers are there between 601 and 700?

701, 709, 719, 727, 733, 739, 743, 751, 757, 761, 769, 773, 787, 797

How many prime numbers are there between 701 and 800?

809, 811, 821, 823, 827, 829, 839, 853, 857, 859, 863, 877, 881, 883, 887

How many prime numbers are there between 801 and 900?

907, 911, 919, 929, 937, 941, 947, 953, 967, 971, 977, 983, 991, 997

How many prime numbers are there between 901 and 1,000?

Euclid (around 300 BCE) proved that there are an infinite number of primes. Graph the data above. Do you notice any pattern?