Rational and Irrational Numbers: Name:_____

A ***rational number*** is a real number that can be written as a ratio of two integers (fraction). This includes integers, decimals, mixed numbers, the square root of square numbers, and repeating decimals.

An ***irrational number*** is a real number that cannot be written as a ratio of two integers. They are non-terminating, non-repeating decimals. i.e. the decimal will go on forever without a repeating pattern.

Irrationality of $\sqrt{n}$ Theorem: if n is an integer that is not a square number, then $\sqrt{n}$ is irrational.

Any repeating decimal can be written as a rational number (fraction.)

ex. 1 Rename $0.\overline{3}$ as a rational number.

a. Let: $1x = 0.\overline{33}$

b. *Multiply both sides of the equation by 10* $10x = 3.\overline{33}$

c. *Subtract:* $\begin{array}{rl} -x &= -0.\overline{33} \\ \hline 9x &= 3 \end{array}$

d. *Solve the equation* $x = \frac{3}{9} = \frac{1}{3}$

e. *There for $0.\overline{3}$ is a rational number* $0.\overline{3} = \frac{1}{3}$

ex. 2 Rename $0.2\overline{17}$ as a fraction.

a. Let: $1x = 0.2\overline{17}$

b. Write equitant equations using powers of 10:

$$\begin{aligned} 1x &= 0.2\overline{17} \\ 10x &= 2.\overline{17} \\ 100x &= 21.7\overline{17} \\ 1000x &= 217.\overline{17} \end{aligned}$$

c. *Subtract 2 of the equations above that will result in the repeating decimal being removed:*

$$\begin{array}{rl} 1000x &= 217.\overline{17} \\ -10x &= -2.\overline{17} \\ \hline 990x &= 215 \end{array}$$

d. *Solve the equation* $x = \frac{215}{990} = \frac{43}{198}$

e. *There for $0.2\overline{17}$ is a rational number* $0.2\overline{17} = \frac{43}{198}$

Rational and Irrational Numbers: Name:_____

A. Write the following numbers as a fraction.

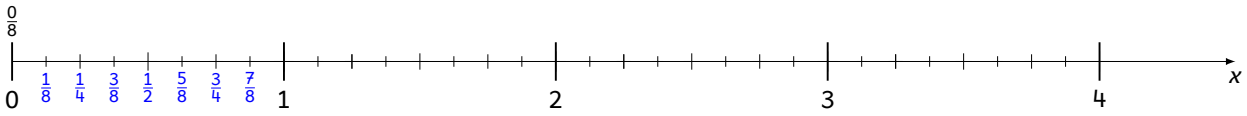
1. -6 = ***2.*** 0.25 = ***3.*** $3\frac{1}{5} =$ ***4.*** $\sqrt{16}$ =

B. Identify the following numbers as rational (R) or irrational (I).

6. π _____ ***7.*** $\sqrt{16}$ _____ ***8.*** $0.\overline{14}$ _____ ***9.*** $\sqrt{5}$ _____

10. 0.1313313331333 ... _____ ***11.*** $\frac{\sqrt{2}}{5}$ _____ ***12.*** $\frac{\sqrt{7}}{\sqrt{7}}$ _____

13. Place the numbers above (6-12) on the number line below.



D. Rewrite each repeating decimal as a fraction.

14. $x = 0.\overline{5}$ ***15.*** $x = 0.\overline{14}$ ***16.*** $n = 0.\overline{145}$

17. $n = 0.\overline{9}$ ***18.*** $x = 0.3\overline{6}$ ***19.*** $a = 0.23\overline{45}$

Rational and Irrational Numbers: Name:_____

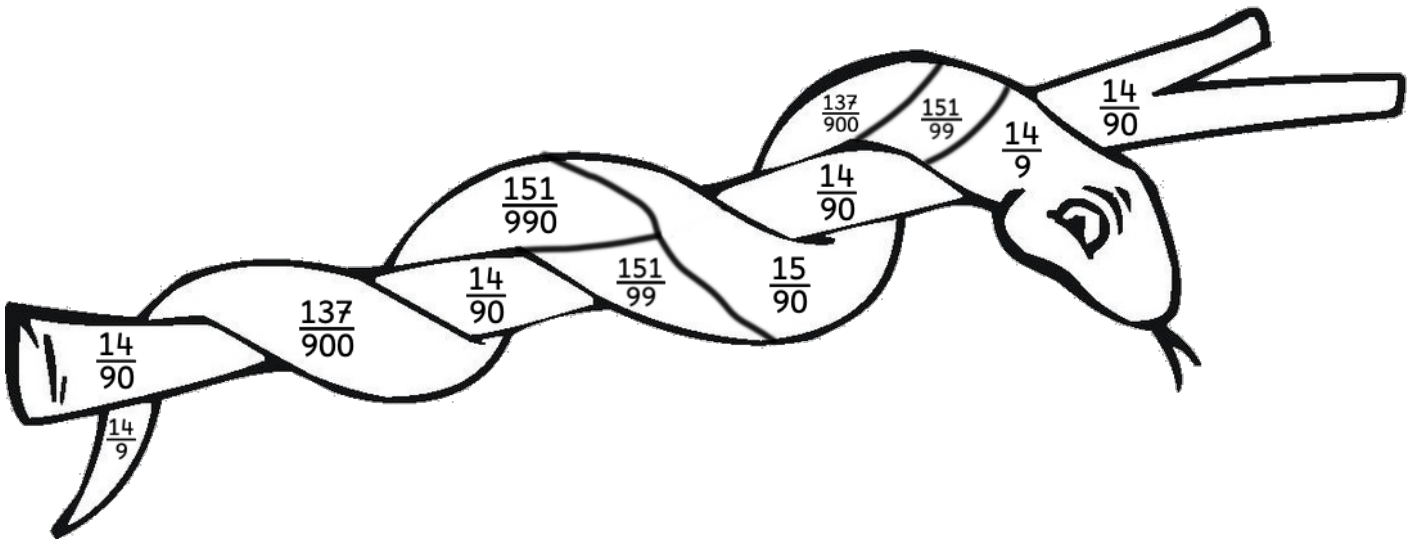
20. *Introduction to Proofs:* Show that $0.\overline{7}$ is a rational number (that it can be written as a fraction.)

STATEMENT	EXPLAIN EACH STEP
$\boldsymbol{x = 0.\overline{77}}$	Given
$\boldsymbol{10x = 7.\overline{77}}$	
$\boldsymbol{10x = \ \ 7.\overline{77}}$ $\boldsymbol{-x = -x}$	
$\boldsymbol{10x = \ \ 7.\overline{77}}$ $\boldsymbol{-x = -0.\overline{77}}$ $\boldsymbol{9x = 7}$	Since $-x = -0.\overline{77}$, we can replace $-x$ with $-0.\overline{77}$
$\boldsymbol{\frac{9x}{9} = \frac{7}{9}}$	
$\boldsymbol{x = \frac{7}{9}}$	
$\boldsymbol{0.\overline{77} = \frac{7}{9}}$	
There for $\boldsymbol{0.\overline{77}}$ *is a rational number*	Rational numbers and numbers that can be written as a fraction.

Rational and Irrational Numbers: Name:_____

Convert the repeating decimal to a fraction. *Show all your work.* (Answers are not simplified.) Check your answer by finding it on the snake. Then color that space the color of the problem.

green = $1.\overline{5}$	brown = $0.1\overline{5}$
orange = $0.1\overline{52}$	blue = $0.15\overline{2}$
red = $1.\overline{52}$	yellow = $0.1\overline{6}$



Rational and Irrational Numbers: Name:_____

Fun with other Rational and Irrational Properties

Given:

- ***Irrationality of $\sqrt{n}$ Theorem*** tells us that if n is an integer that is not a square number, then $\sqrt{n}$ is irrational. So $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$, $\sqrt{6}$, $\sqrt{7}$, $\sqrt{8}$ *... are irrational.*
- *The sum of an irrational number and a rational number is always irrational number.*
- The product an irrational number with any nonzero rational number is always irrational.

What can we figure out?

Read through the and complete the examples below, and then fill in the statements with always, sometimes, never. For all problems: *a, b, c, and d are rational numbers, $b \neq 0$ and $d \neq 0$*

1. The sum of two rational numbers is _____ rational.

1.a. $\frac{a}{b} + \frac{c}{d} =$

2. The product of two rational numbers is _____ rational.

2.a. $\frac{a}{b} * \frac{c}{d} =$

3. An rational number raised to an rational number is _____ rational.

3.a. $a^2 = a * a$ *and* $9^{1/2} = \sqrt{9} =$ _____ *and* $5^{1/2} = \sqrt{5}$

4. The sum of two irrational numbers is _____ irrational.

4.a. $3\sqrt{5} + 2\sqrt{5} =$_____ *and* $7 + 2\sqrt{2} + {}^{-}2\sqrt{2} =$_____

5. The product of two irrational numbers is _____ irrational.

5.a. $\sqrt{5} * \sqrt{7} =$_____ *and* $\sqrt{2} * \sqrt{8} =$_____

6. An irrational number raised to an irrational number is _____ irrational.

6.a. $\left(\sqrt{2}^{\sqrt{2}}\right)^{\sqrt{2}} = \sqrt{2}^{\sqrt{2} * \sqrt{2}} = \sqrt{2}^{2} =$ _____