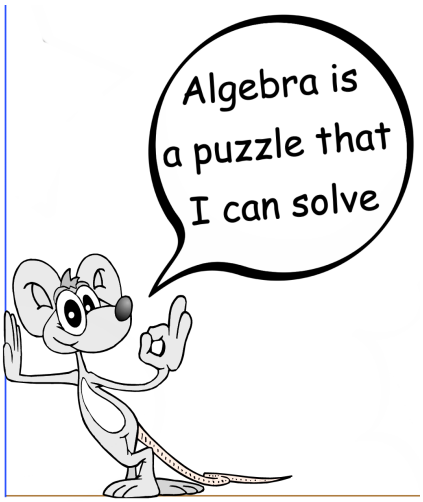
Ch. 1.1

What is Algebra?

Algebra is more than just a series of equations, it is also a way of thinking logically about the world. Algebra is a process that lets us explore the abstract relationships between different things and then provides us with a way to describe those relationships.

We use variables and other symbols as tools to help us express this abstraction in a way that is easier to work with.



Science Says -- Everyone Can Learn Algebra!

- Believe in yourself. Effort, motivation, and resilience will lead to success.
- People with persevere despite setbacks are more successful.
- Visualizing and making connections will strengthen your brain.
- When you learn how to solve new kinds of problems, your brain grows new neurons and you get smarter.
- Intelligence is learned and not fixed. Your brain actually adapts and rewires itself when you learn new things, and it can do so relatively quickly.

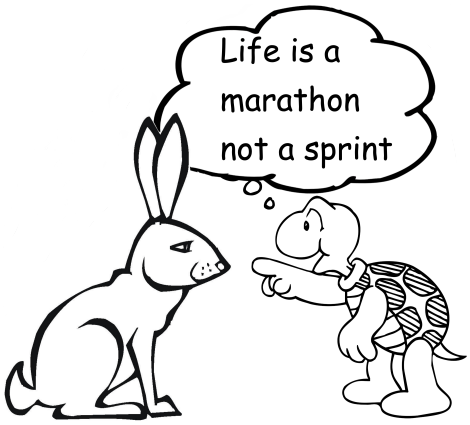
You are a Mathematician, and Mathematicians:

- take on challenges and learn from them
- accept that if they can't do it yet, with effort they will
- grow their understanding of new concepts step by step
- explain their solutions in a way that's easy for others to follow
- know that they will sometimes be wrong, but are willing to try again
- realize that depth is more important than speed

https://www.frontporchmath.com/topics/algebra/language-of-algebra/what-is-algebra-video/

Ch. 1.1

What to do if you get stuck:



- make sure you understand the problem
- draw a picture
- if you can't solve the problem, try to solve a simpler version of the problem
    - use easier numbers
    - keep simplifying the problem until it becomes clear
- work backwards and ask yourself:
    - what am I trying to find out?
    - what information do I need to know?
    - how do I find that information?
- ask questions and seek input from others
    - questions help our understanding of the material and can lead us to new insights
    - talk about what you have tried, discuss what you can do next
- remind yourself that you have overcome setbacks in the past, and you can figure this out

Sources:

Dr. Carol Dweck, Growth Mindset Videos,

The power of believing that you can improve, TED Talk by Dr. Carol Dweck,
https://www.ted.com/talks/carol_dweck_the_power_of_believing_that_you_can_improve?language=en

Growth Mindset by Carol Dweck (animated book summary) - Growth Mindset and Fixed Mindset , Nov. 12014
https://www.youtube.com/watch?v=EyIF5VUOJc0

Solve Problems Like a Mathematician: What you've learned in school is more powerful than you think.
Richard Dancsi, "Dear Life, Please Improve", Psychology Today, Posted Apr 27, 2020
https://www.psychologytoday.com/us/blog/dear-life-please-improve/202004/solve-problems-mathematician

Grit: The Power of Passion and Perseverance – Angela Duckworth, April 2013
https://www.ted.com/talks/angela_lee_duckworth_grit_the_power_of_passion_and_perseverance

The Neuroscience of Growth Mindset and Intrinsic Motivation – Betsy Ng,
PMC-US National Library of Medcine, NIH; Jan. 26, 2018
https://www.ncbi.nlm.nih.gov/pmc/articles/PMC5836039/

1.1 ***Map Coloring*** Name: _____

Although Map Coloring is a topic of Graph Theory, the basic idea is simple and can demonstrate how mathematicians think.

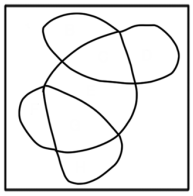
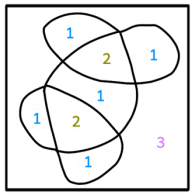
1. Start with a question:

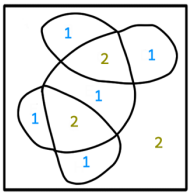
"Have you ever colored in a pattern and wondered what is the smallest number of different colors needed so that no two sections that share a common edge are the same color?"

For example: What is the smallest number of colors you can use to color this drawing?

Hints: Using numbers to represent each color lets you change your mind.

Also, remember that having a corner (point) in common is ok.

ex.  → 1st try 

→ 2nd try 

2. It is ok to make mistakes. The processes you use matter.

While trying to find out how many colors are needed and then proving their conjectures (educated guesses), mathematicians went down many wrong paths, but what they learned along the way led to other advancements in mathematics.

3a. Draw it out:

Color in the maps on the worksheet 1-A, using the smallest number of colors you can.

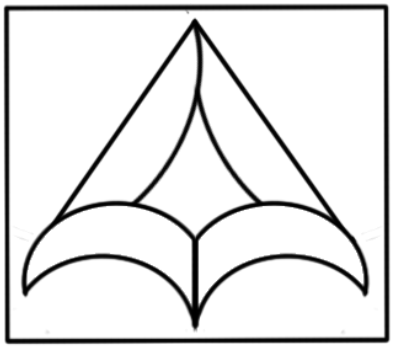
3b. Then create your own "map." Color your designs using the smallest number of colors you can.

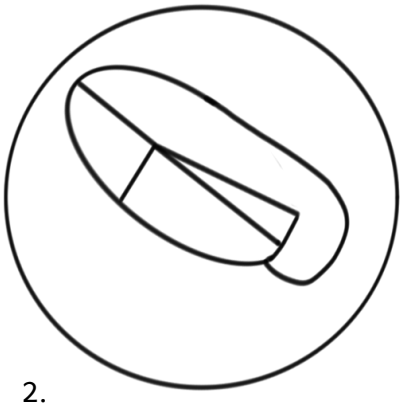
https://www.frontporchmath.com/topics/numbers-patterns/discrete-math/four-color-theorem-map-coloring/

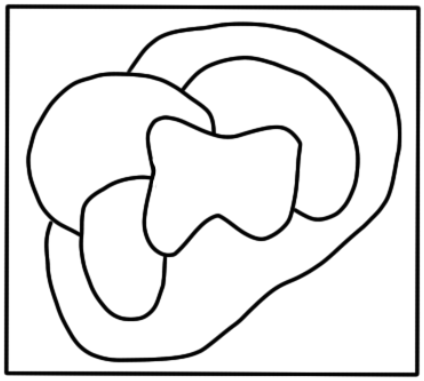
1.1 ***Map Coloring*** Name: _____

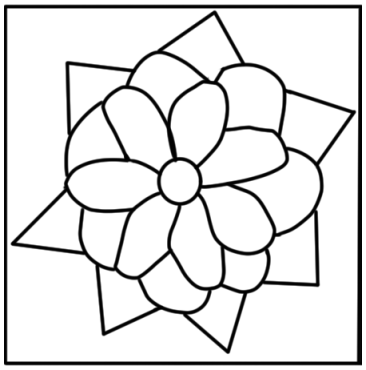
Worksheet 1-A

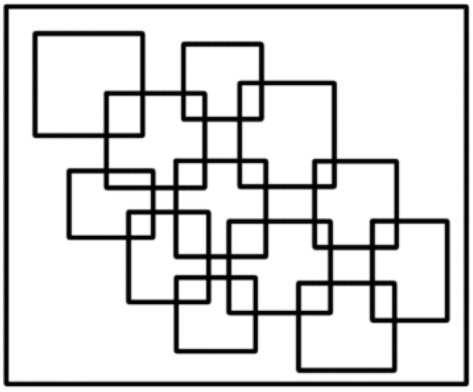
Color in the maps below, using the smallest number of colors you can. *(You may want 2 copies of this sheet)*

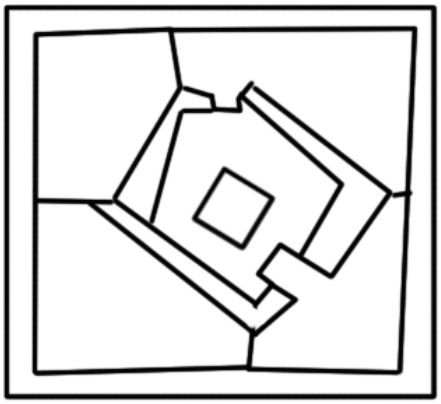
1. 

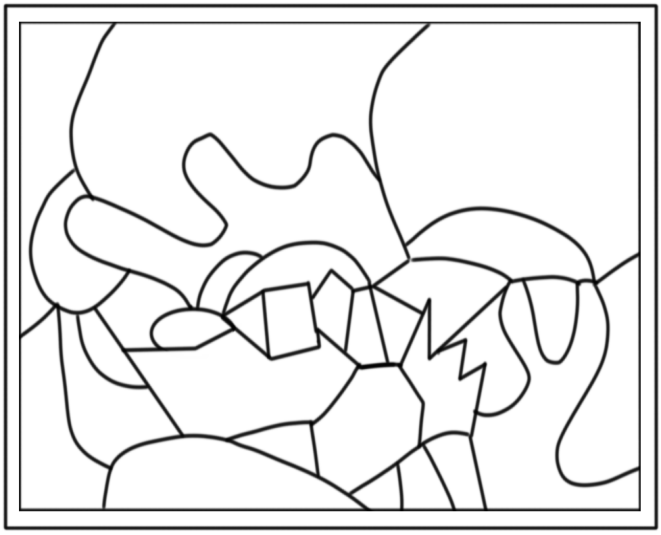
2. 

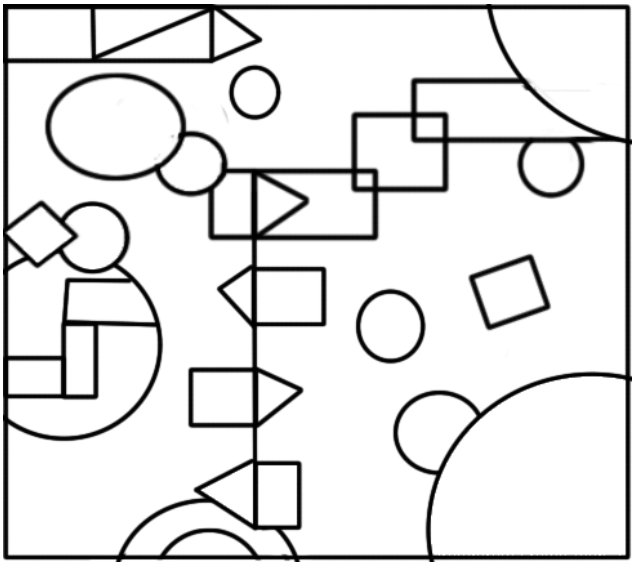
3. 

4. 

5. 

6. 

7. 

8. 

https://www.frontporchmath.com/topics/numbers-patterns/discrete-math/four-color-theorem-map-coloring/

1.1 ***Map Coloring*** Name: _____

4. Talk to Others

Discuss the different strategies people used to minimize the number of colors used.
Compare your work. Could you have used fewer colors? Try redoing the design that required the most colors and see if you can color it using fewer colors.

How we talk to others about math

- I agree/disagree with you because ...
- Can you explain your steps to me?
- What strategy did you use?
- Is there another way to solve this problem?
- My first step was ...
- I still have a question about ...

5. **Make a conjecture** of how many colors one needs to color any pattern.

I think all "maps" can be colored using _____ colors.

6. **Prove the conjecture: Speed is not the most important thing.**

In 1852 Frederic Guthrie (a map maker and mathematician) noticed he only needed 4 colors to color any map and wondered if this was true for all maps. It seemed like a simple idea, but sometimes simple ideas can be very hard to prove. The Four-Color Map Theorem, was not proven until 1976, 124 year later. (This was the first time a computer was used to prove a major mathematical theorem.)

7. **Challenge yourself**

Redo a graph that took more than four colors, or try and create a graph that requires 5 colors.

8. Map Coloring Game

Here is a two-person game that may give you insights into map coloring.

Playing the game:

The first player draws a region.
The second player colors it and draws a new region.
The first player colors it and adds a third region.
The players go back and forth until somebody gets stuck and has to use a fifth color.

When the game is over: Redraw the regions (without the colors) and show that it can be colored in only four colors.

https://www.frontporchmath.com/topics/numbers-patterns/discrete-math/four-color-theorem-map-coloring/